Table 4.8. **Calculation of χ^2 on Data from Investigation 3 (Table 3.2)**

Results	*O*	*E*	*O* – *E*	$(O-E)^2$	$(O-E)^2/E$
Heads on both coins					
Heads on one, tails on the other coin					
Tails on both coins					
Totals	48			χ^2 =	
				Degrees of freedom:	
				P range:	
				I **accept/reject** the hypothesis that the data approximate the expected ratio.	

A. Practice Problems

Use a calculator to calculate χ^2 in the two examples that follow. The χ^2 values you should obtain are given. If the value you obtain does not agree with the value given, repeat the calculation until agreement is reached. Notice how the χ^2 calculated is interpreted.

1. In a cross involving *Drosophila melanogaster*, an F_2 population included 272 flies with long (normal) wings and 60 flies with dumpy wings. Calculate χ^2 and fill in the blanks in the following table. Do these results approximate a 3:1 ratio?

Phenotype	*O*	*E*	*O* – *E*	$(O-E)^2$	$(O-E)^2/E$
Normal	____	____	____	____	____
Dumpy	____	____	____	____	____
Totals	____	____			

χ^2 = ________

Instructor's calculation of χ^2 = 8.4980

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate a 3:1 ratio? ______________________

c. What is the probability that the deviations are due to chance alone?

____

2. In a dihybrid **test cross** the following results were obtained: *A–B–*, 121; *A–bb*, 107; *aaB–*, 115; and *aabb*, 93. Do these data approximate a dihybrid test cross ratio with independent assortment? Complete the following table and answer the related questions.

Phenotype	O	E	O − E	$(O-E)^2$	$(O-E)^2/E$
A–B–	121				
A–bb	107				
aaB–	115				
aabb	93				
Totals				χ^2 =	

Instructor's calculation of χ^2 = 4.0367

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate a dihybrid test cross ratio with independent assortment?

____

c. What is the probability that the deviations are due to chance alone?

____

B. Assigned Problems

On the following pages are six χ^2 problems to be solved using a calculator. Your instructor will assign one or more of these problems to be submitted at a designated time.

1. The following are dihybrid F_2 data. Do these data support the hypothesis of independent assortment? Complete the table, calculate χ^2, and answer the questions based on your calculations.

Phenotype	O	E	$O - E$	$(O - E)^2$	$(O - E)^2 / E$
A–B–	470				
A–bb	144				
aaB–	161				
aabb	41				
Totals				$\chi^2 =$	

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate the dihybrid F_2 ratio with independent assortment? ______

c. What is the probability that the deviations are due to chance alone?

______

d. Complete the following table for the data given. Determine whether each gene pair is behaving according to Mendel's first law, giving a 3:1 ratio. Note that since F_2 data are being considered in this problem, one might reasonably expect each individual trait to behave according to Mendel's law of segregation, giving a 3:1 ratio.

Hypothesis	χ^2 Value	P Value	Accept or Reject Hypothesis
3 *A–* : 1 *aa*			
3 *B–* : 1 *bb*			

2. The following are dihybrid test cross data. Do these data approximate the ratio one would expect for independent assortment? Complete the table, calculate the χ^2, and answer the questions related to your calculations.

Phenotype	O	E	$O - E$	$(O - E)^2$	$(O - E)^2 / E$
A–B–	151				
A–bb	37				
aaB–	38				
aabb	148				
Totals				$\chi^2 =$	

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate the dihybrid test cross ratio with independent assortment?

____

c. What is the probability that the deviations are due to chance alone?

____

d. Complete the following table for the data given in this problem. Note that the data in this problem are dihybrid test cross data. Now determine whether each gene pair is behaving individually as you would expect in a monohybrid test cross. Each trait considered individually should be expected to approximate a 1:1 ratio as a consequence of Mendel's law of segregation.

Hypothesis	χ^2 Value	*P* Value	Accept or Reject Hypothesis
1 *Aa* : 1 *aa*	______	______	______
1 *Bb* : 1 *bb*	______	______	______

e. In view of the χ^2 tests obtained for each trait individually, how might you account for the dihybrid test cross ratio obtained?

____

3. Some studies show that the following are the approximate frequencies of the various ABO blood types in the U.S. white population: 41% A, 9% B, 3% AB, and 47% O. Do the data given in the following table represent a satisfactory sample of the U.S. white population? Complete the table, calculate χ^2, and answer the questions based on your calculations. Calculate *E* values to the nearest one-hundredth (0.01).

Phenotype	*O*	*E*	*O* – *E*	$(O - E)^2$	$(O - E)^2/E$
A	260	______	______	______	______
B	55	______	______	______	______
AB	18	______	______	______	______
O	311	______	______	______	______
Totals	______	______		χ^2 =	______

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that the data given represent the distribution of blood types in the U.S. white population?

____

c. What is the probability that the deviations are due to chance alone?

____

d. Determine the percentage of the sample that each blood type constitutes, and record this information in the following spaces:

A ______ %; B ______ %; AB ______ %; O ______ %.

4. Probability theory and the binomial expansion show that if you were to sample families consisting of four children, 1/16 of these families would consist of 4 boys, 4/16 would consist of 3 boys and 1 girl, 6/16 of 2 boys and 2 girls, 4/16 of 1 boy and 3 girls, and 1/16 of 4 girls. Do the data in the sample given in the next table approximate this expectation? Complete the table, calculate χ^2, and answer the questions based on your calculations.

Family Sex Ratio	O	E	O – E	$(O - E)^2$	$(O - E)^2/E$
All boys	497	______	______	______	______
3 B : 1 G	1863	______	______	______	______
2 B : 2 G	2699	______	______	______	______
1 B : 3 G	1779	______	______	______	______
All girls	410	______	______	______	______
Totals	______	______			

$\chi^2 =$ ______

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate the ratio given? ______________________________

c. What is the probability that the deviations are due to chance alone?

____

d. Determine whether the overall ratio of boys to girls in the above data is consistent with the hypothesis of a 50:50 sex ratio. Remember that each family included in the table consists of four children; for example, 497 families consisted of four boys, 1863 families consisted of three boys and one girl, and 2699 families consisted of two boys and two girls. Calculate χ^2 for these data by completing the following table:

Sex	O	E	O – E	$(O - E)^2$	$(O - E)^2/E$
Male	____	____	____	____	____
Female	____	____	____	____	____
Totals	____	____		$\chi^2 =$	____

e. **Accept/reject** ____________ ; *df* = ____________ ; *P* = ____________

f. Calculate the ratio of boy to girls; record here: ____________________

____

g. How have biologists explained sex ratio data such as those observed in this problem?

____

____

5. If both parents have the genotype *Aa* and if they produce a family of four children, the binomial expansion shows the probabilities of obtaining families of different phenotypic combinations, as follows: 81/256 all *A–* ; 108/256 3 *A–* : 1 *aa*; 54/256 2 *A–* : 2 *aa* ; 12/256 1 *A–* : 3 *aa* ; and 1/256 all *aa*. A sample was obtained of a large number of families consisting of four children in which both parents were known to be *Aa*. Do these data approximate the expected distribution? Complete the following table, calculate χ^2, and answer the related questions.

Family Ratios	O	E	O – E	$(O - E)^2$	$(O - E)^2/E$
All *A–*	1401	____	____	____	____
3 *A–* : 1 *aa*	1799	____	____	____	____
2 *A–* : 2 *aa*	903	____	____	____	____
1 *A–* : 3 *aa*	235	____	____	____	____
All *aa*	14	____	____	____	____
Totals	____	____		$\chi^2 =$	____

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate the ratio suggested above? ______________________

c. What is the probability that the deviations are due to chance alone?

____

d. Note that if the mating *Aa* × *Aa* is made, the theoretically expected ratio among the offspring is 3 *A*– : 1 *aa*. How many families in the table have four children in the expected 3:1 ratio? ______________

What percentage of the total is this? ______________

e. What is the theoretically expected percentage of the families that should have four children in a perfect 3:1 ratio? ______________

f. How do you account for this low frequency of the expected results?

____

6. Binomial expansion tells us that in families of 5 children you may expect 1/32 of the families to consist entirely of boys, 5/32 to consist of 4 boys and 1 girl, 10/32 to consist of 3 boys and 2 girls, 10/32 to consist of 2 boys and 3 girls, 5/32 to consist of 1 boy and 4 girls, and 1/32 to consist entirely of girls. Do the data on such families of five children given in the following table approximate the distribution suggested? Complete the table, calculate χ^2, and answer the related questions.

Family Sex Ratio	O	E	$O - E$	$(O - E)^2$	$(O - E)^2/E$
All boys	48	____	____	____	____
4 B : 1 G	219	____	____	____	____
3 B : 2 G	404	____	____	____	____
2 B : 3 G	371	____	____	____	____
1 B : 4 G	177	____	____	____	____
All girls	29	____	____	____	____
Totals	____	____			

$\chi^2 =$ ______________

a. In interpreting this χ^2 value you have ______ degrees of freedom.

b. In this case do you **accept/reject** the hypothesis that these data approximate the ratio given above? ____________________

c. What is the probability that the deviations are due to chance alone?

____

d. Determine whether the overall ratio of boys to girls in the data is consistent with the hypothesis of a 50:50 sex ratio. Remember that each family included in the table consists of five children; for example, 48 families consisted of five boys, 219 families consisted of four boys and one girl, and 404 families consisted of three boys and two girls. Calculate χ^2 for these data by completing the following table:

Sex	O	E	$O - E$	$(O - E)^2$	$(O - E)^2/E$
Male	______	______	______	______	______
Female	______	______	______	______	______
Totals	______	______			______

χ^2 = ______

e. **Accept/reject** ________ ; df = ________ ; P = ________

f. Calculate the ratio of boys to girls; record here: ____________________

____

C. Some Cautions When Using Chi-Square

Some authorities recommend that a certain adjustment be made if the sample in any class is relatively small. For your purposes, however, the calculated value for χ^2 without this correction is satisfactory. Statisticians also suggest that calculating χ^2 is inappropriate when the sample size in any class is less than five. **With certain exceptions, χ^2 calculations must be based on numerical frequencies and not on percentages or ratios.**

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Note: All references cited in Investigation 3 are also pertinent to this investigation.